The Interrelation of Surface Effects and Acceleration in the Burnout Heat Flux Problem

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Most theories of pool-boiling burnout which are based on hydrodynamic instability models indicate that burnout heat flux is proportional to the local acceleration raised to the 0.25 power. The nature of the theories is such that surface effects should alter neither the magnitude of the burnout heat flux nor the extent to which the burnout heat flux is affected by acceleration.

Data collected by Bernath (*1*) and results reported recently (*2, 3*) have clearly shown that for heat transfer surfaces of practical sizes, surface effects can alter the burnout heat flux, a fact which is in direct contradiction to the hydrodynamic instability theories. Since surface effects are important in the burnout problem, it appears that the extent to which acceleration affects the burnout heat flux should also depend on the surface nature. Data presented in this communication clearly indicate that surface effects are definitely interrelated with acceleration in the burnout heat flux problem.

APPARATUS

The heaters employed were graphite strips (0.3 in. wide, 2.5 in. long with 2-in. between voltage taps, insulated on all but one face) and a platinum wire (0.063 in. in diameter 3 in. long, 2.3 inches between voltage taps). The heaters were mounted at a radius of 2.3 to 2.4 in. in a centrifuge (Figure 1). (Radius was measured to the axis of the wire and to the uninsulated face of the strip.) The heaters were mounted with their long axes parallel to the center line of rotation so that the centripetal acceleration was normally away from the graphite strip.

The acceleration vector was normally away from the inside half of the platinum wire but normally toward the outer half. However, it has been shown (*4*) that in such cases burnout always originates at the inside of the wire, that is, the side from which the acceleration vector is directed. Hence, burnout data from the wire and the strips are comparable.

A fast response potentiometer was used to measure the d.c. voltage across the heaters. With this instrument, it was possible to interrupt the power the instant burnout started and to save the heater. Moreover, because of the rapid interruption of power, the heater surfaces showed no evidence of alteration. The same heater was used for a number of tests, and frequent checks were made at a reference condition (zero revolutions per minute of the centrifuge) to insure that no variation in the surface had occurred.

Fluids employed in the tests were distilled water, distilled water with an additive (see reference *4* for specifications) and 200 proof ethanol. The fluids were maintained at the saturation temperature during the tests by power supplied from the test heaters (*4*). All tests were conducted at pressures close to atmospheric. There were slight pressure increases caused by

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Unsteady Diffusion with First-Order Reaction

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Outlined below is a method for simplification of the solution of diffusion problems involving irreversible first-order reactions. It permits writing the concentration profiles for reactive systems in terms of those for nonreactive systems of the same geometry and similar boundary conditions. The method is valid where fluid flow patterns and concentration boundary conditions are time independent and where the assumptions of constant density and mass diffusivity are acceptable. The useful results of this development are summarized in Equations (31) through (33) and represent a generalization of an earlier development by P. V. Danckwerts (*1*).

Considered here is a binary system, composed primarily of species *A* and *B* and defined by the following equations:

$$\frac{\partial c_A}{\partial t} + (\mathbf{v} \cdot \nabla c_A) = \mathcal{D}_{AB} \nabla^2 c_A + k\, c_A \quad (1)$$

$$c_A(\mathbf{r}, 0) = c_0(\mathbf{r}) \quad (2)$$

$$c_A(\mathbf{s}, t) = c_s(\mathbf{s}) \quad (3)$$

$$\mathbf{v} = \mathbf{v}(\mathbf{r}) \quad (4)$$

$$\rho = \text{constant} \quad (5)$$

These equations describe unsteady state mass transfer in a system of time-independent velocity profile and boundary conditions for which the mass diffusivity $\mathcal{D}_{AB}$, and total mass density, ρ, are constant.

It is the purpose of this communication to show how the concentration profile for this system may be determined from that of one for which $k = 0$ but is otherwise identical to that described by Equations (1) to (5). The problem may be split into two simpler ones by defining

$$c_A = c_{A1} + c_{A2} \quad (6)$$

with c_{A1} and c_{A2} described by

$$\frac{\partial c_{A1}}{\partial t} + (\mathbf{v} \cdot \nabla c_{A1}) = \mathcal{D}_{AB} \nabla^2 c_{A1} + k\, c_{A1} \quad (7)$$

$$c_{A1}(\mathbf{r}, 0) = c_0(\mathbf{r}) \quad (8)$$

$$c_{A1}(\mathbf{s}, t) = 0 \quad (9)$$

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Axial diffusion and pressure drop of liquids in porous media, Stahel, E. P., and C. J. Geankoplis, **A.I.Ch.E. Journal, 10,** No. 2, p. 174 (March, 1964).

Key Words: A. Mass Transfer-8, Packed Bed-8, Random Media-8, Porous Solid-8, Water-5, Liquid-5. B. Frequency Response-10, Diffusion-8, Axial Diffusion-8, Eddy Diffusion-8, Mixing-8, Pressure Drop-8, Dynamic Response-10, Analogy-9.

Abstract: Experimental data were obtained on axial diffusion coefficients for water flowing through porous media with frequency response techniques. The random porous media used were made by pouring Wood's metal over a packed bed of salt crystals, allowing the metal to solidify, and leaching out the salt.

D_L values were found to vary as $U^{1.46}$. These values of D_L were found to be much higher than the values found by others for packed beds. The D_L and pressure drop in the porous media suggest a possible analogy between momentum transfer and D_L. Pressure drop data were correlated by Ergun's equation at higher velocities.

Heat transfer in the critical region—temperature and velocity profiles in turbulent flow, Wood, R. D., and J. M. Smith, **A.I.Ch.E. Journal, 10,** No. 2, p. 180 (March, 1964).

Key Words: Flow-1 Heat-1, Inconel-1, 10, Temperature Profile-2, Velocity Profile-2, Heat Transfer Coefficient-2, Critical Region-5, Carbon Dioxide-5, Critical Pressure-6, Critical Temperature-6, Heat Flux-7, Bulk Temperature-7, Wall Temperature-7, Heat Transfer-8, Tube-10.

Abstract: Temperature and velocity profiles and local heat transfer coefficients were measured for turbulent flow of carbon dioxide in a tube at 1,075 lb./sq. in. abs. (critical pressure-1,071 lb./sq. in. abs.). The results indicate a severe flattening of the radial temperature profiles, a maximum in the velocity profile between tube axis and wall, and a maximum in heat transfer coefficient, when the bulk fluid temperature passes through the transposed critical temperature.

Turbulent motion and mixing in a pipe, Lee, Jon, and R. S. Brodkey, **A.I.Ch.E. Journal, 10,** No. 2, p. 187 (March, 1964).

Key Words: Diffusivity-6, Distance-6, Injection Velocity-6, Concentration-7, Decay-8, Mixing-8, Turbulent-8, Dye Solution-9, Intensity-9, Scalar-9, Spectrum-9, Water System-9, Fiber Optic Light Probe-10, Hot Film Anemometer-10, Pipes-10.

Abstract: Turbulent mixing of a dye solution injected at the center of a pipe was studied by means of a new light probe developed for the measurement of concentration fluctuations. The measurements of mean concentration and intensity of concentration fluctuations were made both along the axial distance and across the pipe. The phenomenological approximations of Corrsin and of Beek and Miller could be used for the practical problem of predicting the decay of the intensity of the concentration fluctuations or mixing in the pipe geometry studied.

Heat transfer characteristics of fluids moving in a Taylor system of vortices: Parts I and II. Ho, C. Y., J. L. Nardacci, and A. H. Nissan, **A.I.Ch.E. Journal, 10,** No. 2, p. 194 (March, 1964).

Key Words: Heat Transfer-8, Fluids-5, Concentric Cylinders-, Taylor Vortices-, Thermal Boundary Layers-8, Prandtl Number-6, Groshoff Number-6, Heat Transfer Coefficients-8.

Abstract: Heat transfer coefficients, in the form of Nusselt numbers, increase when fluids rotating in the gap between concentric cylinders exceed a critical parameter known as the **critical Taylor number.** A theoretical approach for Prandtl number equal to 1 successfully gives this increase for a short range of Taylor numbers above the critical value. An empirical relation is developed which gives the heat transfer coefficients for the Taylor system for a wide range of property values and speeds of rotation. Experimental results of this work are combined with those of previous researches to demonstrate these points.

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$$\frac{\partial c_{A2}}{\partial t} + (\mathbf{v}\cdot\nabla c_{A2}) = \mathcal{D}_{AB}\,\nabla^2 c_{A2} + k\,c_{A2} \qquad (10)$$

$$c_{A2}(\mathbf{r}, 0) = 0 \qquad (11)$$

$$c_{A2}(s, t) = c_s(\mathbf{r}) \qquad (12)$$

These two sets of equations may be solved separately, beginning with the solution of Equations (7) through (9) by introduction of a new variable defined by

$$c_{A1} = g\,e^{kt} \qquad (13)$$

Equations (7) through (9) may now be rewritten in terms of g as follows:

$$\frac{\partial g}{\partial t} + (\mathbf{v}\cdot\nabla g) = \mathcal{D}_{AB}\,\nabla^2 g \qquad (14)$$

$$g(\mathbf{r}, 0) = c_0(\mathbf{r}) \qquad (15)$$

$$g(s, t) = 0 \qquad (16)$$

Thus, g is simply the solution to Equations (7) to (9) for $k = 0$, and Equation (13) is part of the desired result.

Next consider Equations (10) through (12), and begin here by taking the Laplace transform with respect to time. After rearrangement one obtains

$$(p-k)\cdot\overline{c}_{A2}\,(p, \mathbf{r}) = \mathcal{D}_{AB}\,\nabla^2\,\overline{c}_{A2} - (\mathbf{v}\cdot\overline{c}_{A2}) \qquad (17)$$

$$\overline{c}_s = c_s/p \qquad (18)$$

The corresponding expressions for f, the solution to Equations (10) through (12) with $k = 0$, are

$$p\cdot\overline{f}(p, \mathbf{r}) = \mathcal{D}_{AB}\,\nabla^2\overline{f} - (\mathbf{v}\cdot\nabla\overline{f}) \qquad (19)$$

$$\overline{f}_s = c_s/p \qquad (20)$$

The problem now is to put Equations (19) and (20) into the same form as (17) and (18).

Begin by writing $(p-k)$ for p in Equations (19) and (20):

$$(p-k)\cdot\overline{f}(p-k) = \mathcal{D}_{AB}\,\nabla^2\overline{f}\,(p-k) - (\mathbf{v}\cdot\nabla f(p-k)) \qquad (21)$$

$$\overline{f}_s(p-k) = c_s/(p-k) \qquad (22)$$

Now multiply both sides of Equations (21) and (22) by $(p-k)/p$ to obtain

$$\left[\left(\frac{p-k}{p}\right)\cdot\overline{f}(p-k)\right]\cdot(p-k) = \mathcal{D}_{AB}\nabla^2\left[\left(\frac{p-k}{p}\right)\cdot\overline{f}\,(p-k)\right] - \left[\mathbf{v}\cdot\nabla\left(\left(\frac{p-k}{p}\right)\cdot\overline{f}(p-k)\right)\right] \qquad (23)$$

$$\left[\left(\frac{p-k}{p}\right)\cdot\bar{f}_s\ (p-k)\right] = c_s/p \qquad (24)$$

Clearly (23) and (24) are now entirely analogous to Equations (17) and (18). One may therefore write

$$\bar{c}_{A2}(p) = \left(\frac{p-k}{p}\right)\cdot\bar{f}_s\ (p-k) =$$

$$\left(1-\frac{k}{p}\right)\bar{f}_s\,(p-k) \qquad (25)$$

Next recognize that

$$\bar{f}(p-k) = \bar{F}(p) \qquad (26)$$

where

$$F = f\,e^{kt} \qquad (27)$$

and that

$$\frac{1}{p}\bar{F} = \bar{I} \qquad (28)$$

where

$$I = \int_o^t F\,dt \qquad (29)$$

One may then take advantage of Equations (26) and (28) in inverting Equation (25) to obtain

$$c_{A2} = f\,e^{kt} - k\int_o^t f\,(\mathbf{r},\tau)\,e^{k\tau}\,d\tau \qquad (30)$$

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ress in the nuclear-reactor field. Of importance, particularly to those giving instruction in reactor theory, are the many fresh new problems given at the end of nearly every chapter. A high percentage of these draw from real experience as reported in the literature and also draw the student to the literature in a critical and rewarding way.

In sixteen chapters the author sets the context, develops the simpler methods of analysis, gradually works his way to the more sophisticated techniques, and concludes with firm-design orientation. Throughout, one finds adequate emphasis on relating theory and experiment. This reviewer found particularly commendable the author's treatment of neutron diffusion in nonmultiplying media where graphical comparisons give an excellent feel for the applicability of infinite media solutions to certain finite media problems.

Inevitably, perhaps, a few errors or misstatements have not been caught. Some of the more important ones, which may create false impressions in inexperienced minds include: an early statement that "natural uranium is used as the fuel material for most thermal reactors," whereas in fact virtually all but the large plutonium production reactors in this country are heavily committed to the use of uranium enriched in the U-235 isotope; an equation (9-24) improperly suggesting itself, without qualification, as a general solution for the time-dependent neutron density problem with time-varying reactivity; and, finally, a tabular value of the multiplication factor for an infinite homogeneous "optimum" mixture of natural uranium and light water in excess of 1.0. In addition to these specifics, this reviewer suggests that the first of the two chapters on reactor dynamics might have imparted a little more of the feel of the subject than it did. It is also somewhat disappointing to see nothing in the final chapter on reactor dynamics on the subject of xenon instability. These comments should not dissuade the reader, however, from the overall impression of excellence.

Professor Isbin reveals his chemical engineering background in a few places which meaningfully tie the material to other areas of more traditional chemical-engineering activity. For the most part, however, the book is directed to the serious student of applied physics and should be readily comprehensible, indeed invaluable, to him.

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INFORMATION RETRIEVAL

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Hydrates at high pressures: part I. methane-water, argon-water, and nitrogen-water systems, Marshall, Donald R., Shozaburo Saito, and Riki Kobayashi, **A.I.Ch.E. Journal, 10,** No. 2, p. 202 (March, 1964).

Key Words: Hydrates-1, Clathrates-1, Inclusion Compounds-2, High Pressure Apparatus-10, Phase Behavior-8, Gas-Liquid-Solid Equilibria-8, Nitrogen-Water-1, Argon-Water-1, Methane-Water-1, Pressure Generation-10, Water-Nitrogen-1, Water-Argon-1, Water-Methane-1.

Abstract: Gas hydrate equilibrium curves were determined for argon, methane, and nitrogen hydrates at high pressures by an indirect method based on the difference in gas content of the solid hydrate and the liquid phase. An experimental apparatus has been built to study gas hydrates at pressures up to 100,000 lb./sq.in.abs. To date the hydrate equilibrium curves for the methane-water, argon-water, and nitrogen-water system have been determined to the vicinity of 60,000 lb./sq.in.abs.

Adiabatic, evaporating, two-phase flow of steam and water in horizontal pipe, Pike, R. W., and H. C. Ward, **A.I.Ch.E. Journal, 10,** No. 2, p. 206 (March, 1964).

Key Words: Annular Flow-1, Evaporating Flow-1, Flashing Flow-1, Vapor-Liquid Flow-1, Flow-1, Concurrent Flow-1, Vaporizing Flow-1, Water-1, 4, Two-Phase Flow-1, 10, Steam-2, 4, Pressure Drop-7, Critical Flow-7, Void Fraction-7, Horizontal Pipe-10, Pipes-10.

Abstract: The adiabatic, evaporating, two-phase flow of steam and water in horizontal pipe including the critical flow was successfully described by an annular flow model which included different phase velocities, fluid acceleration, wall and interface shear forces, and mass and energy transfer between the phases. Experimental data including void fractions showed the theoretical equations and described the complex flow to ± 10% when radial temperature gradients were small. Design charts based on the numerical solution of the theoretical equations are presented for rapid evaluation of the flow variables for the system steam-water including critical flow.

Single-particle studies of ion exchange in packed beds: cupric ion-sodium ion system, Rao, M. Gopala, and M. M. David, **A.I.Ch.E. Journal, 10,** No. 2, p. 213 (March, 1964).

Key Words: A. Exchanging-8, Ion Exchange-8, Cation Exchange-8, Copper-9, Sodium-9, Beds-10, Packed-, Concentration-6, Rate-6, Flow-, Rates-8, Exchanging-, Coefficients-8, Mass Transfer-8, j-Factor-10, Diffusivities-8, Internal-, Constant-, Variable-, Models-10, Mathematical-. B. Exchangers-9, Ion Exchangers-9, Dowex 50W-X8-9, Particles-10, Beads-10, Single-, Homogeneity-8, Radioisotopes-10, Tracers-10.

Abstract: Single particles of Dowex 50W-X8 were used to study exchange of copper for sodium in packed beds, with sodium-22 tracer, for 0.01 N to 4 N solutions and a wide range of flow rates. External mass transfer coefficients for dilute solutions were correlated by the j-factor method. Internal-diffusion controlled data were interpreted equally well by constant interdiffusivities and by equations embodying electric field effects. Data where both internal and external diffusional resistances were important were interpreted by matching with curves computed numerically and including the variable internal-diffusivity model. External mass transfer coefficients thus found for concentrated solutions were not correlated by the j-factor method.

The mechanism of liquid-phase resistance to gas absorption in a packed column, Vivian, J. Edward, and C. Judson King, **A.I.Ch.E. Journal, 10,** No. 2, p. 221 (March, 1964).

Key Words: A. Absorption-8, Mass Transfer-8, Packed Column-10, Helium-1, Oxygen-1, Carbon Dioxide-1, Propylene-1, Air-4, Water-4, Diffusivity-6, Temperature-6, Mass Transfer Coefficient-7. B. Penetration Theory-9, Surface Renewal-9, Absorption-8, Packed Column-10. C. Diffusivity-8, Diaphragm Cell-10, Helium-1, Hydrogen-1, Oxygen-1, Carbon Dioxide-1, Propylene-1, Water-4.

Abstract: The desorption of helium, hydrogen, oxygen, carbon dioxide, and propylene from water into air was studied at five different flow conditions in a packed tower. The mass transfer coefficient was found to vary as the 0.5 power of liquid-phase diffusivity, in agreement with the predictions of penetration theory. The indicated mechanism is based on molecular diffusion processes and is independent of eddy processes during a given surface lifetime.

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This completes the solution.

One may now write

$$c_A(\mathbf{r}, t) = c_{A1} + c_{A2} = g\, e^{kt} + f\, e^{kt} - k \int_0^t f\,(\mathbf{r}, \tau) e^{k\tau}\, d\tau \quad (31)$$

This is the desired answer. The second and third terms on the right side of this equation are the same as those in Danckwerts' solution, but it is shown by the above development that this solution may be extended to systems in which there is steady fluid motion. The first term in the right side is new and extends the Danckwerts solution to systems of finite initial concentration.

Equation (31) may be put into a frequently more convenient alternate form:

$$c_A(\mathbf{r}, t) = g e^{kt} + \int_0^t e^{k\tau} \frac{\partial}{\partial\tau} f(\mathbf{r}, \tau)\, d\tau \quad (32)$$

Here advantage has been taken of the fact that $f(r, 0) = 0$.

The above procedure may also be used to prove that Equations (31) and (32) remain valid if Equations (3) and (12) are replaced by

$$-(\mathbf{n} \cdot \mathcal{D}_{AB}\, \nabla c_A)|_s = k_c\, (c_{As} - c_{Af}) \quad (33)$$

Here k_c, a mass transfer coefficient, and c_{Af} must be time independent, while c_{As}, the boundary concentration, may vary with time. The term $\mathbf{n}$ is an outwardly directed unit normal vector at the system boundary, and $|_s$ refers to conditions at the boundary.

NOTATION

c_A = concentration of solute species A, moles l^{-3}

$\mathcal{D}_{AB}$ = mass diffusivity of A in the solution, $l^2 t^{-1}$

f = solution to Equations (10) to (12) for $k = 0$, assumed available

g = solution to Equations (7) to (9) for $k = 0$, assumed available

k = reaction rate constant, t^{-1}

$\mathbf{r}$ = position vector describing location of all points in system considered, l

$\mathbf{s}$ = position vector describing location of all points on system boundary, l

t = time

$\mathbf{v}$ = fluid velocity vector, lt^{-1}

∇ = gradient operator, l^{-1}

∇^2 = Laplacian operator, l^{-2}

LITERATURE CITED

1. Danckwerts, P. V., *Trans. Far. Soc.*, 47, 1014-1023 (1951).